Int. J. Heat Mass Transfer. Vol. 13, pp. 1331–1338. Pergamon Press 1970. Printed in Great Britain

A TURBULENT HEAT FLUX METER AND SOME MEASUREMENTS OF TURBULENCE IN AIR FLOW THROUGH A HEATED PIPE

P. J. BOURKE and D. J. PULLING
Atomic Energy Research Establishment, Harwell, Berkshire, England

(*Received* 11 *July* 1969 *and in revised form* 18 *November* 1969)

Abstract—A technique has been developed for determining local heat flux from simultaneous measurements of the velocity and temperature fluctuations in a turbulent fluid. The errors to which it is subject have been reviewed and its accuracy has been estimated from preliminary tests in air flow through a heated pipe. The results provide further data for the eddy diffusivity for heat and new data for the correlation between velocity and temperature fluctuations.

NOMENCLATURE

A,	hot wire constant, see equation (3);
B,	hot wire constant, see equation (3);
c_p,	specific heat at constant pressure;
E_T,	cold wire d.c. voltage;
e_T,	cold wire deviation from mean voltage;
E_V,	hot wire d.c. voltage;
e_V,	hot wire deviation from mean voltage;
I,	cold wire current;
k,	thermal conductivity;
Q_{tur},	heat flux by eddy diffusion;
R,	cold wire resistance;
R_0,	cold wire resistance at 0°C;
r,	pipe radius;
T_A,	air temperature;
t,	deviation from mean air temperature;
T_W,	hot wire temperature;
U,	axial velocity;
u,	deviation from mean axial velocity;
U^*,	friction velocity;
v,	radial velocity;
α,	thermal coefficient of resistivity;
H,	eddy diffusivity for heat;
ρ,	density of air;
$'$,	r.m.s. value;
$\bar{\ }$,	time average.

1. INTRODUCTION

In connection with some work on heat transfer to CO_2 near its critical state, a previous paper [1] discussed the interest in rates of heat transfer by turbulent eddy diffusion within the fluid. It suggested the possibility of obtaining the local turbulent heat flux from simultaneous measurements of the turbulent fluctuations in velocity and temperature, using the relationship

$$Q_{tur} = \rho C_p vt \qquad (1)$$

which applies provided the specific heat per unit volume is constant. The present paper describes the technique which has been developed for making these measurements and which is applicable in normal gases and liquids as well as in near critical fluids.

For the velocity measurements, hot wire anemometry was chosen as the only method likely to be successful in following the turbulent fluctuations. For the temperature measurements both thermocouples and resistance thermometers were considered. The latter can be made smaller than the former (10^{-6} m dia. resistance wire compared with the 5×10^{-5} m dia. of the smallest practical thermocouple junction) and were therefore chosen for their lesser thermal inertia and faster response to temperature fluctuations. Probes carrying both anemometer and thermometer wires have been made.

To test these probes and to show the data which they can obtain, a few measurements were made of the radial distributions of heat flux and mean values and fluctuations of velocity and temperature in the turbulent core of air flowing through a long, uniformly heated pipe. Values of the heat flux were calculated, firstly from the known wall flux and the established mean velocity profile and secondly from the velocity and temperature fluctuations. The agreement between these values is considered good enough to prove the principle of the technique. The conditions of these tests did not allow its accuracy to be precisely established but the errors to which is is generally subject have been identified and are discussed here.

In making these tests with air, the radial gradient of mean temperature was obtained and has been used to calculate the eddy diffusivity for heat. The values presented here are appreciably lower than previous measurements [2, 3]. The results also give the correlation between the velocity and temperature fluctuations which does not appear to have been previously measured in flow through a heated pipe.

2. PROBE AND ELECTRONICS

The probe used to carry the anemometer and thermometer wires, was a modified DISA miniature X hot wire probe. The X arrangement of hot wires was chosen to allow measurement of radial velocity in a pipe. Two extra pins (sewing needles of 5×10^{-4} m dia.) were cemented to the ceramic stem of the probe at opposite ends of a diameter normal to the plane of the X. Their points were level with the points of the longer hot wire pins. Wollaston wire, silver with a platinum core, was used as the resistance thermometer (the cold wire) and was soldered to the needle points. A length of about $7{\cdot}5 \times 10^{-4}$ m in the middle of this wire was electrolytically etched to expose the platinum with a resistance of about 50 Ω. Photo micrographs of this length showed it to be approximately circular with a diameter of about 10^{-6} m.

The whole array of wires lay within a volume of dimensions less than 10^{-3} m with the cold wire up stream of the hot wires and therefore out of their thermal wakes. Being at right angles to the hot wires the wake of the thin cold wire was not expected to interfere seriously with the hot wire measurement of velocity. This assumption was checked experimentally on occasions when the cold wire broke without detectable change in the hot wire readings.

A block diagram of the electronic circuitry used with this probe is shown in Fig. 1. The hot wires were operated normally by two DISA 55 A 01 anemometers. The difference between the "turbulence outputs" of these instruments was formed to give the radial component of turbulent velocity in the usual way. This difference was then fed, after appropriate amplification, to one input of a wide band (up to 50 Kc/s) Burr–Brown multiplying circuit.

A battery in series with a high resistance was used to maintain a constant current of 0·5 mA through the cold wire. The voltage across it was amplified and its mean value, after smoothing, was measured by a digital voltmeter to give the mean temperature. The r.m.s. value of its fluctuating a.c. component was measured by a Dawe r.m.s. voltmeter. This fluctuating component was fed to the second input of the multiplier.

The output of the multiplier thus gave the product of the voltage fluctuations due to turbulent fluctuations in the radial velocity and temperature. The mean value of this product was measured by a damped galvanometer. Its instantaneous value together with those of the radial velocity and temperature voltages, were monitored on a multi-beam oscilloscope.

3. OTHER APPARATUS

The probe was tested in air flow at atmospheric pressure through a stainless steel pipe of 5 m length, $4{\cdot}16 \times 10^{-2}$ m i.d. and $1{\cdot}4 \times 10^{-3}$ m wall thickness. The probe was

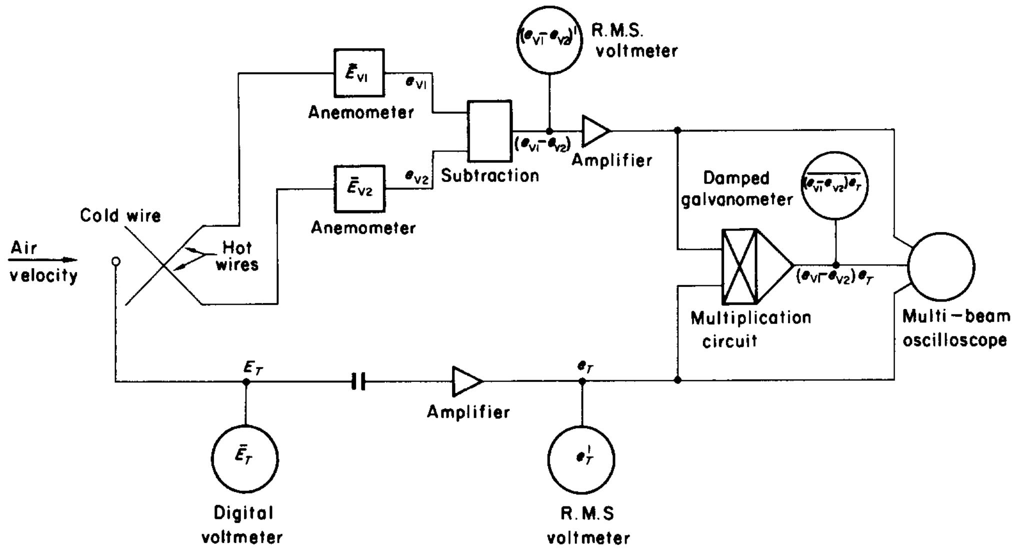

FIG. 1. Block diagram of electronics.

inserted axially into the pipe to put the wires two diameters up stream from its open end and could be traversed across a diameter.

The pipe was thermally insulated along its whole length and a 2·44 m length from its outlet was electrically heated by passing current along its wall. This heating could produce uniform heat flux up to $1{\cdot}43 \times 10^3$ W/m^2 from the inner surface of the pipe to the air flow. The 2·44 m length, giving a heated length to diameter ratio of 55, was adequate to establish fully developed velocity and temperature profiles at the position of the wires [4]. The wall temperature at this position was measured by a thermocouple.

Air could be passed through the pipe at velocities up to 12·2 m/s giving a Reynolds number of 30000. A pre-heater before the test section was provided to give control of the air inlet temperature at values up to 100°C.

4. SOURCES OF ERROR

4.1 *Calibration of cold wire*

The cold wire was calibrated in the range 15–100°C against a standard thermocouple with isothermal conditions and 9·15 m/s air velocity in the test section. This calibration gave its thermal coefficient of resistance α as 0·00378 per °C without systematic error, using the relationship

$$E_T = RI = R_0 I(1 + \alpha T_A). \tag{2}$$

Using this calibration, it was estimated that the wire could indicate changes in air temperature for a given velocity to an accuracy of 0·05°C.

An error can however arise with change in velocity, due to the heating effect of the current through the cold wire. This raises the wire temperature above that of the ambient air. Hence, change in air velocity and therefore also in the heat transfer coefficient of the wire, changes both the wire temperature and its calibration against air temperature. This error can be quantified from the heat transfer coefficient for cross flow over a cylinder [5], the wire dimensions and the heat release determined from the wire current and voltage. Calculation indicated that for the velocities in the present work, the wire temperature did not exceed the air temperature by more than 0·1°C. This calculation was checked experimentally by increasing the cold wire current to 2 mA which gave a measured rise in wire temperature of about 1·5°C, in agreement with a calculated rise of 1·6°C. The error in indicated air temperature over the range of velocity in the present experiments, was calculated to be appreciably less than 0·05°C and must have had negligible effect on the accuracy of the results.

4.2 *Response of cold wire*

To allow this technique to be successful, the cold wire must respond rapidly to temperature fluctuations. Its response time to a step change in air temperature can be calculated from its dimensions, specific heat and heat transfer coefficient. (The effects of conduction within the wire and the thermal capacity of the air surrounding it are negligible.) For the velocities in the present work, calculation showed that following a step change in temperature the wire indicated half this change within less than 3×10^{-5} s. The most rapid temperature fluctuations observed on the oscilloscope had periods of 5×10^{-4} s or longer and therefore the error due to the cold wire not responding instantaneously to these fluctuations is thought to have been negligible.

4.3 *Calibration of hot wires*

The thermal coefficient of resistance, 0·00395 per °C of the hot wires was measured in the same way used for the cold wire. From this, their resistance at 290°C was calculated and they were always operated with this resistance constant.

They were calibrated with isothermal conditions in the test section for velocities from 3·05 to 12·2 m/s and air temperatures from 15 to 60°C which covered the range of operating conditions in the present work. During calibration, the wires were positioned on the centreline of the pipe and the local axial velocity was calculated from the measured volume flow rate and the velocity profiles for the Reynolds number of the flow [6]. With the wires at 45° to the axial direction, their calibration fitted the usual relationship

$$E_V^2 = [A + B(\bar{U}/\sqrt{2})^{\frac{1}{2}}](T_W - T_A). \tag{3}$$

The calibrations of the two wires were the same within experimental error.

A more accurate form of equation (3), such as that suggested by Hinze [7], to allow for the sensitivity of wires to the velocity component parallel to them, should be used. The error due to neglecting this sensitivity is however small compared with other errors in the present results and equation (3) has therefore been used here for its simplicity.

When the two wires, 1 and 2, are operated in turbulent conditions with heat transfer, their voltages fluctuate with the fluctuations of both the velocity and temperature of the air. These fluctuations are given by

$$e_{V1} = \left(\frac{u+v}{\sqrt{2}}\right)\sqrt{2}\,\frac{\partial E_V}{\partial \bar{U}} + t\,\frac{\partial E_V}{\partial T_A} \tag{4}$$

$$e_{V2} = \left(\frac{u-v}{\sqrt{2}}\right)\sqrt{2}\,\frac{\partial E_V}{\partial \bar{U}} + t\,\frac{\partial E_V}{\partial T_A}. \tag{5}$$

Subtraction gives the radial velocity

$$v = \frac{e_{V1} - e_{V2}}{2}\,\frac{\partial \bar{U}}{\partial E_V} \tag{6}$$

in the usual way because the temperature terms cancel, provided the two wires are identical and have the same temperature sensitivity $\partial E_V/\partial T_A$. In the present experiments with the r.m.s. temperature fluctuations of the order of 3°C, the magnitude of the two terms in the right hand sides of equation (4) and (5) were approximately equal. Hence the error in equation (6), due to the assumption that the temperature terms cancelled on subtraction, was comparable with that usually made in assuming that both wires have the same velocity sensitivity $\partial E_V/\partial \bar{U}$. The radial velocity was therefore estimated from equation (6) using the value of $\partial \bar{U}/\partial E_V$ obtained from the isothermal calibrations at an air temperature equal to the mean temperature indicated by the cold wire.

This estimate is still subject to error in that it assumed $\partial \bar{U}/\partial E_V$ does not vary with fluctuations of the air temperature. In fact, equation (3) shows that $\partial \bar{U}/\partial E_V$ must be a function of $(T_W - T_A)$. Hence the magnitude of this further error is related to $t'/(T_W - T_A)$. In the present experiments with values of this ratio about 1 per cent it was shown from the calibrations to have produced errors in the value of v of about 3 per cent.

4.4 *Spatial resolution*

Errors may also have arisen in the present preliminary experiments in a pipe of $4{\cdot}16 \times 10^{-2}$ m dia., because the wire array was not small compared with the microscale of turbulence. The dimension of the former was about $7{\cdot}5 \times 10^{-4}$ m whereas the latter was estimated from the criterion given by Hinze [7] to be about 5×10^{-4} m. Hence the wires cannot have responded to the small scale fluctuations in velocity and temperature. It is not possible to quantify the error due to this source but the accuracy of the heat flux measurements, given below as 8 per cent, suggests that most of the heat transfer was produced by the large scale fluctuations.

4.5 *Overall accuracy*

It is not possible to give a general figure for the accuracy of this technique in measuring turbulent heat flux. The above review of possible errors does however suggest that the worst source of inaccuracy is usually likely to be failure to match the sensitivities of the two hot wires. For small values of the ratio $t'/(T_W - T_A)$, say less than 3 per cent, the accuracy should be about the same as that with which X hot wires can normally measure radial velocity, in practice probably about 10 per cent with carefully wired probes [8]. This estimate is in agreement with the results below. For larger values of the ratio $t'/(T_W - T_A)$ the accuracy will deteriorate due to only approximate cancellation of the temperature terms in subtraction of equations (4) and (5) and to variations in $\partial \bar{U}/\partial E_V$ in equation (6) with fluctuations in air temperature.

RESULTS

5.1 *Typical experimental data*

Traverses with the probe in the heated pipe were made with heat flux from the wall from $2{\cdot}6 \times 10^2$ to $13{\cdot}0 \times 10^2$ W/m² and with mass flow rates from $4{\cdot}76 \times 10^{-3}$ to $15{\cdot}9 \times 10^{-3}$ kg/s giving Reynolds numbers from 7700 to 25000. Values of the wall heat flux were calculated both from the electrical power input to the test section and from the mass flow and bulk temperature rise of the air. These values agreed to within 5 per cent.

A typical set of results for a wall heat flux of $13{\cdot}0 \times 10^2$ W/m² and a Reynolds number of 14900 is given in Fig. 2 in which the mean axial

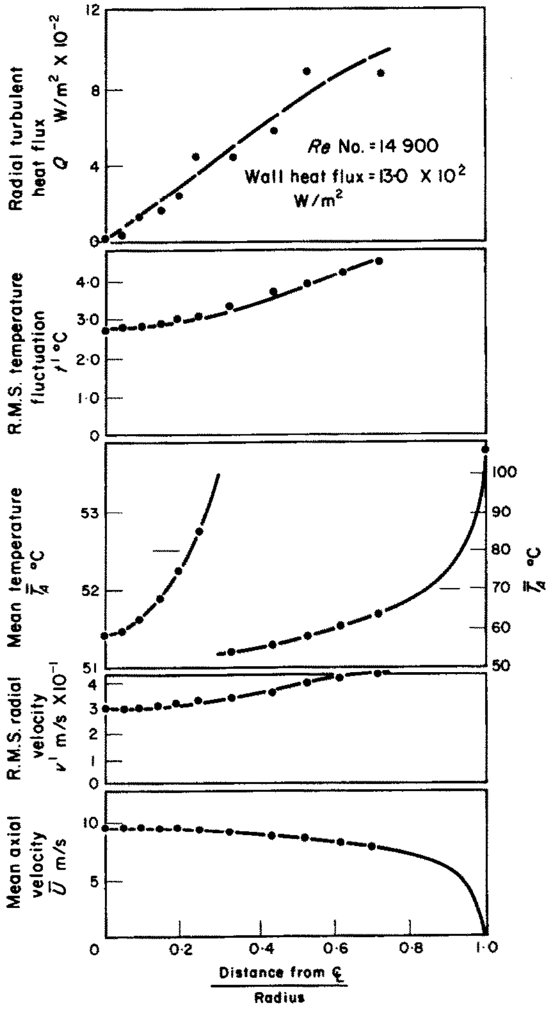

FIG. 2. Typical experimental data.

velocity, the r.m.s. radial velocity, the mean air temperature and the r.m.s. temperature fluctuation are plotted against radius. Values of the mean radial turbulent heat flux were calculated from the relationship

$$\bar{Q}_{\text{tur}} = \rho C_p \overline{v t}$$

which, after substitution of equations (2), (3) and (6), becomes

$$\bar{Q}_{\text{tur}} = \rho Cp \tfrac{1}{2} \overline{(e_{V1} - e_{V2}) e_T} \frac{\partial \bar{U}}{\partial E_V} \frac{\partial T_A}{\partial E_T}$$

$$= \rho Cp \overline{(e_{V1} - e_{V2}) e_T} \frac{2 E_V \bar{U}}{E_V^2 - A(T_W - T_A)} \times \frac{1}{R_0 l \alpha} \qquad (7)$$

and are also plotted in Fig. 2.

These values were checked independently as follows. At the 53:1 heated length to diameter ratio of the probe position the axial mean velocity profile was fully developed and the axial mean temperature gradient was invariant with axial and radial position. Hence this profile could be used to determine the radial distribution of total heat flux by both molecular conduction and turbulent diffusion. The conduction flux was calculated from the radial gradient of mean temperature and subtracted from the total to give the diffusion flux. The values thus determined are plotted as the broken line in Fig. 2 for comparison with the values obtained from the velocity and temperature fluctuations and shown by the experimental points. The accuracy of the latter is given by the 'points' showing no significant deviation from the calculated line and their random scatter about it having a standard deviation of about 8 per cent.

5.2 *Eddy diffusivity for heat*

From the results obtained with a wall heat flux of 13·0 × 10^2 W/m^2 and a Reynolds number of 25 100 the eddy diffusivity was calculated using the relationship

$$Q_{\text{tur}} = \varepsilon_H \frac{\partial T}{\partial r}. \qquad (8)$$

The values obtained are compared with similar measurements by Sleicher [2] and Johnk and Hanratty [3] in Fig. 3. In this graph the data are presented as total diffusivity divided by the pipe radius and the friction velocity, this being the method of presentation suggested [9] and used [3] to allow comparison of data from pipes of differing radius. The present data are about 25 per cent lower than the previous data over most of the turbulent core of the flow.

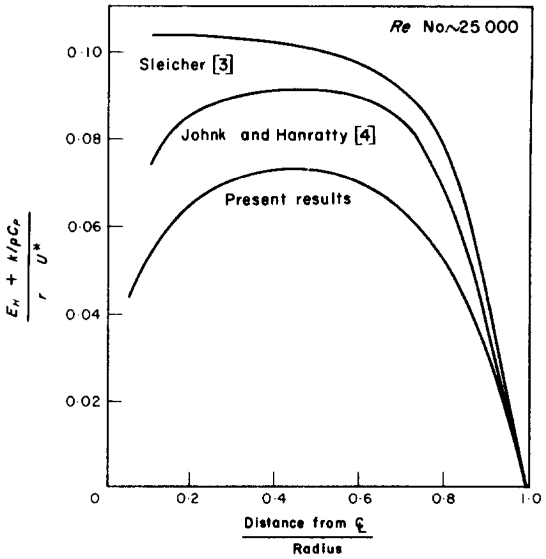

FIG. 3. Comparison of present values of total diffusivity with previous measurements.

In all three of these investigations the accuracy of determining ε_H was limited by errors in the measurement of the radial temperature gradient which was low and difficult to determine accurately in the turbulent core of the flow. The previous workers [2, 3] deliberately obtained their data with small temperature differences of less than 10°C between the pipe wall and centre line and, hence, very low gradients, because they measured the velocity profiles with isothermal conditions only. In their experiments these

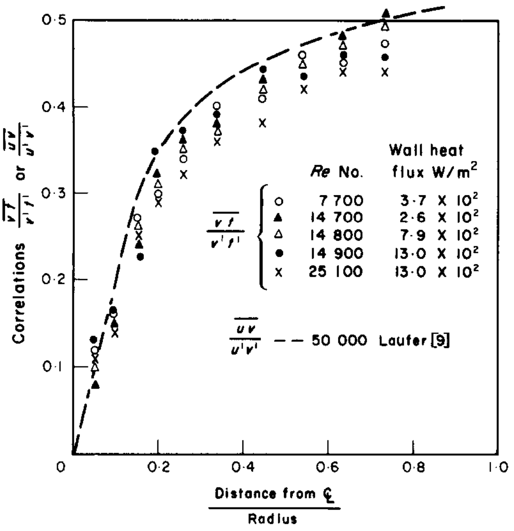

FIG. 4. Comparison of correlations for transfer of heat and momentum.

profiles were needed to determine the heat flux and they therefore wished not to increase the temperature differences sufficiently to change the velocities. In the present experiments with direct measurement of heat flux, temperature differences up to 50°C were used and therefore the present measurements of temperature gradient and diffusivity may be more accurate.

5.3 *Velocity temperature correlation*

The present experimental data also give the correlation between the fluctuations in velocity and temperature, defined as $\overline{v\,t}/v'\,t'$. Values obtained from all the data are plotted in Fig. 4. These values are insensitive to both heat flux and Reynolds number over the ranges investigated.

This correlation provides an alternative method, to that using eddy diffusivity, for characterising turbulent heat transfer. The latter depends on the assumption that the motion of turbulent eddies is dependent only on the local gradients of mean velocity and temperature. The validity of this assumption for large eddies has been questioned [10, 11]. Hence the velocity temperature correlation may be a more accurate characteristic of turbulent heat transfer than eddy diffusivity. It also has the advantage that, unlike diffusivity [2, 3] it does not vary with Reynolds number and it may be easier to measure accurately because it does not depend on the temperature gradient.

If the basic mechanisms for the turbulent transfer of heat and momentum are similar, this correlation for heat transfer should be comparable with the similar correlation for momentum transfer i.e. $\overline{u\,v}/u'\,v'$. The latter correlation [12] which is also insensitive to Reynolds number is plotted as the broken line in Fig. 4. The ratio of $\overline{v\,t}/v'\,t'$ to $\overline{u\,v}/u'\,v'$ is nearly unity at all radial positions. (The approximately 5 per cent difference is barely significant, allowing for the experimental accuracy.) This ratio is therefore suggested as an alternative to the turbulent Prandtl number the values of which are uncertain [13], for quantifying the analogy between the transfer of heat and momentum.

6. CONCLUSIONS

1. A technique has been developed for measuring local heat flux and turbulence in the presence of temperature gradients. It may be useful in investigating the mechanisms of heat transfer and of practical value in situations (e.g. non-uniform conditions, roughened surfaces) in which the transfer cannot be calculated from measurements of the mean velocity and temperature profiles.
2. Measurements of the eddy diffusivity for heat near the centre of a pipe, possibly more accurate than previously available, have been made.
3. Alternative methods, to those using eddy diffusivity, have been suggested for describing turbulent heat transfer and the analogy between heat and momentum transfer.

7. ACKNOWLEDGEMENTS

The authors wish to acknowledge several helpful discussions with Mr. P. Bradshaw and Mr. R. F. Johnson of the National Physics Laboratory and Dr. J. Whitelaw of Imperial College. Mr. D. Benn of the Chemical Engineering Division, AERE, contributed much to the success of the experiments by his skilful wiring of the probes. Mr. L. E. Gill, now of Oxford University, collaborated with the authors in the development of the technique in experiments in supercritical CO_2, to be described elsewhere.

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UN APPAREIL DE MESURE DE FLUX DE CHALEUR EN TURBULENT ET QUELQUES MESURES DE TURBULENCE DANS UN ÉCOULEMENT D'AIR À TRAVERS UN TUYAU CHAUFFÉ

Résumé—Une technique a été établie pour déterminer le flux de chaleur local à partir de mesures simultanées des fluctations de vitesse et de températures dans un fluide turbulent. Les erreurs auxquelles elle est sujette ont été examinées et sa précision a été estimée à partir d'essais préliminaires dans l'écoulement d'air à travers un tuyau chauffé. Les résultats fournissent des renseignements supplémentaires pour la diffusivité turbulente de la chaleur et de nouvelles données pour la corrélation entre les fluctuations de vitesse et de température.

EIN WÄRMESTROMMESSGERÄT FÜR TURBULENTE STRÖMUNGEN UND EINIGE TURBULENZMESSUNGEN IN EINER LUFTSTRÖMUNG DURCH EIN BEHEIZTES ROHR

Zusammenfassung— Es wurde eine Technik zur Bestimmung des lokalen Wärmestroms durch gleichzeitige Messung der Geschwindigkeits- und Temperaturschwankungen in einer turbulenten Strömung entwickelt. Die Fehler dieses Verfahrens wurden geprüft, und die Genauigkeit wurde aus vorläufigen Versuchen in einer Lustströmung durch ein beheiztes Rohr bestimmt. Die Ergebnisse bringen weitere Daten für die turbulente Wärmeübertragung und neue Daten für die Beziehung zwischen den Geschwindigkeits- und Temperaturschwankungen.

ИЗМЕРЕНИЕ ТУРБУЛЕНТНОГО ТЕПЛОВОГО ПОТОКА И СТЕПЕНИ ТУРБУЛЕНТНОСТИ ПРИ ТЕЧЕНИИ ВОЗДУХА ЧЕРЕЗ НАГРЕТУЮ ТРУБУ

Аннотация—Разработан метод определения локального теплового потока на основе одновременного измерения пульсаций скорости и температуры в турбулентном потоке жидкости. Оценены погрешности и точность измерений из предварительных опытов по течению воздуха в нагретой трубе. На основе результатов получены данные по турбулентной диффузии тепла и новые данные по определению корреляций пульсациями скорости и температуры.